A new young animal trade module for CAPRI

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Background and motivation

Since the early stages of CAPRI, net trade of young animal at EU Member states is simulated to close young animal markets at EU level. At later stage, different blocks had been introduced for EU15, EU10, Bulgaria & Romania, Norway, and for Western Balkans. The main aim of the module was to ensure a plausible mix of animal production activities in simulations. The very first tries in the project phase 1996-1999 tried to use a guessed matrix of own and cross-price elasticities in a square system along the lines of a multi-commodity model. However, guessing the elasticities resulting from the regional programming model proved impossible, and the approach was soon given up as the system did not converge. Instead, aggregated versions of the regional programming models at Member State level were defined by aggregating the objective function terms, constraint vectors and the coefficient matrix. The Member states models were then calibrated to the current decision variable levels from the regional models using PMP. Rather larger models resulted from stacking these Member States models together with constraints ensuring market clearing for young animals, and the shadow values of the latter defined the prices used in the next iteration of the regional programming models.

As the number of regions has grown over time due to the regional expansion of the model, the preparation and solution of the young market model has grown considerably. It profited from recent improvements in GAMS as the grid solve option and improvements in CONOPT, but less so then the regional programming models, and, to a certain extent, the global market model, so that in recent version almost 40% of the computing time of an iteration was spend on the young animal markets. That percentage could only be achieved by the application of the grid solve, fixing of small crop activities and feed input coefficients, which let to rather complex GAMS code.

From the methodological side, clearing markets for young animals at EU level without considering transport and transaction costs was also not very inviting. But the layout of model with more then 6000 variables and around 4000 constraints for the EU15 excluded effectively a further expansion. It became obvious that an alternative solution must be searched for.

A square system

The original idea of using a square system of equations where price changes would drive young animal demand and supply in behavioural equations was revitalised again. The difference to the situation in 1996-1999 is the fact that since Heckelei 2002 the functional relation between the supply responses, the quadratic cost function terms used in the regional programming models and the coefficients relating to binding constraints is known:

$$(1) \qquad \frac{\partial \mathbf{x}}{\partial \mathbf{gm}} = \mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{A}'\left(\mathbf{AQ}^{-1}\mathbf{A}'\right)^{-1}\mathbf{AQ}^{-1}$$

Where **x** denotes the decision variables, **gm** the vector of gross margins, **A** the constraint coefficient matrix and **Q** the quadratic terms of the variable costs function of the regional programming models. Accordingly, the supply effects can be calculated from the parameters of the regional programming models, and the animals market cleared as:

$$(2) \qquad \sum_i \left(x_i^0 + \sum_{nj} \Delta p_n IO_{jn} \frac{\partial x_i}{\partial gm_j} IO_{im} \right) = 0 \wedge m$$

Where *m*,*n* are the young animals, *i* the production activities, and **IO** the input output coefficients for young animals. 0 denotes the current solution returned from the regional programming models. The term $\Delta p_n IO_{jn}$ captures the absolute change in the gross margin of the activity j if the price of the young animal type *n* is changed, and $\frac{\partial x_i}{\partial gm_n} IO_{in}$ converts the change in the activity level of activity *i* into a change of supply or demand for young animal animals.

Value added for the global market model

The behavioural functions in the global market model are derived from a non-symmetric normalised quadratic cost function with the price index for non-agricultural products used the normalisation price. Accordingly, the supply functions are linear in the prices of the global market, and the supply effects $\frac{\partial S_x}{\partial p_y}$ for the products in the market model *x*,*y* can be defined by $\sum_{yij} IO_{jy} \frac{\partial x_i}{\partial gm_j} IO_{ix}$. The first term IO_{jy} captures again the effect of a change in price *y* on the gross margin of activity *j*, where as the second term $\frac{\partial x_i}{\partial gm_j} IO_{ix}$ captures the marginal increase of output *x* due to the change in activity level *i*. It is hence possible to update parameters of the supply functions in the market model in each iteration based on the terms used in young animal market models, which promises to speed up and ease solution of the market model.

Implementation in GAMS

The implementation in CAPRI is based on two relatively small programs. “supply\simu_yani_model.gms” defines the square system as a NLP problem with a dummy objective , and the parameters and sets used. “supply\simu_yani_new.gms” first calculated a simplified **A** matrix from the coefficient matrix of the regional models, and aggregates them to Member State level. The simplification consists especially in using fixed feed input coefficients. Arable, grass land, the set-aside constraint, fodder balances and the milk quota are taken into account. During the first iteration, the regional **Q** matrices are inverted, and then summed up to Member State level. Due to updates of the **A** matrix, the term $-\mathbf{Q}^{-1}\mathbf{A}'\left(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}'\right)^{-1}\mathbf{A}\mathbf{Q}^{-1}$ is recalculated for the Member States in each iteration, which requires an additional inversion.

As the Δp_n are only variables in linear system, and currently still defined at the level of country blocks as EU15, the solver defines a solution in less then a second. The calculation of (2) are more demanding, especially the inversion of the **Q** for all regions requires close to one minute for all 252 regions currently covered, but is only necessary once. The remaining calculations take around 5 seconds, so that animal markets can now be solved in about 10%-20% of the previously required time on a powerful four processor machine. Due to the modifications, a typically iteration in CAPRI now requires around 90 seconds if the market model does not require a pre-solve.